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The flow near the bow of a steadily turning ship

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The flow near the bow of a steadily turning ship is analysed using a modified slender-body theory. The rate of change of flow quantities in the longitudinal (x) direction is assumed to be greater than that implied by 'conventional' slender-body theory. As a consequence some features of high Froude number flow are apparent which cannot be predicted by the 'conventional' theory. The modified slender-body theory proposed requires the solution of a two-dimensional Laplace equation (in y and z) but its free-surface condition still involves an x derivative. A Fourier-transform method is used to solve this problem. A simple bow configuration of constant draft is analysed and numerical results for the free-surface elevation are presented.

1. Introduction

In previous work (Hirata 1972*a*) a wing of zero thickness and small aspect ratio was used as a mathematical model to simulate the flow around a steadily turning ship. The shape of the wing was given by the projection of the ship hull on the vertical plane of symmetry and the aspect ratio or draft/length ratio was assumed to be small, say $O(\epsilon)$. Since the angle of attack varies along the length of a ship turning steadily, a camber was added to the wing, which was then assumed to follow a straight path. This kind of model was also used by Fedyayevskiy & Sobolev (1964) but they did not analyse the case of a cambered wing.

In the above-mentioned work the free-surface effects were found to be of higher order (as a consequence of the model adopted). However, it is expected that the free surface has some effect on the flow, especially in the bow region, where characteristics of high Froude number flow can be encountered as described by Ogilvie (1972) and Hirata (1972*b*). If the Froude number can be taken as a measure of the relative magnitude of the inertial and gravitational forces, taking the Froude number to be $O(1)$ as $\epsilon \to 0$ means that neither of these forces dominates the other. This fact seems to be quite true over the major part of the ship length and the usual slender-body model (adopted in Hirata 1972*a*) can be used with accuracy. However, observing the region near the bow, it may be noted that slender-body theory does not describe precisely the flow there, i.e. the ratio of inertia to gravitational forces is no longer $O(1)$ since the effects of water displacements by the moving ship are much greater than the effects of gravity. (In the extreme case, beyond the scope of the present analysis, water spilling occurs.) In order to analyse these different flow characteristics near the ship, a conveniently modified slender-body theory has been introduced. Note that in the

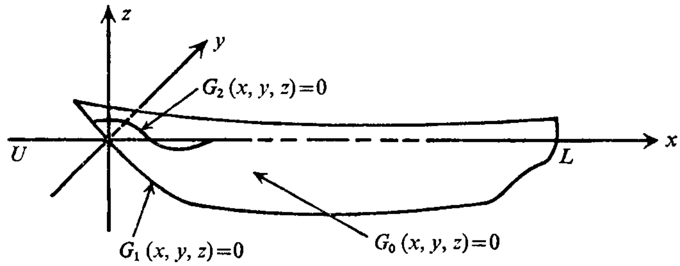

FIGURE 1. The co-ordinate system and definition of body.

usual way one stretches the co-ordinates in the plane normal to the flow by a factor ϵ^{-1}, which is a formal way of saying that the rate of change of the flow quantities in the transverse direction is larger than that in the longitudinal direction. This is the description assumed to be valid over most of the ship. Near the bow, however, one assumes that the rate of change of the flow quantities is greater than near the rest of the ship, and this is accomplished by stretching the longitudinal co-ordinate by a factor of $\epsilon^{-\frac{1}{2}}$, the usual stretching in the transverse direction remaining unchanged. With this unconventional stretching one obtains a condition on the free surface near the bow similar to that used in thin-ship theory, but the continuity of the fluid is still expressed by a two-dimensional Laplace equation. This description is assumed to be valid within a region which extends a distance $O(\epsilon^{\frac{1}{2}})$ from the bow. A simple case of a ship of constant draft is analysed, and the numerical results are given in the form of a plot of the free-surface elevation in the region of the bow.

2. Formulation of the problem

Definitions and assumptions

Let us adopt a co-ordinate system fixed in the ship with origin at the intersection of the bow and the horizontal plane defined by the undisturbed free surface. The x axis is directed aft and the z axis is taken positive upwards; see figure 1.

The wing shape (or ship contour) is defined by

$$G_1(x, y, z) = z - h(x) = 0 \tag{1}$$

and the wing surface (or ship hull) by

$$G_0(x, y, z) = y - b(x) = 0. \tag{2}$$

The ship's draft and beam are assumed to be functions only of the longitudinal co-ordinate, since the introduction of y dependence would introduce complexity into the algebra without bringing any new physical insight into the problem. With regard to their orders of magnitude the following assumptions are made:

$$L = O(1), \quad h(x) = O(\epsilon), \quad b(x) = O(\epsilon^{1+\gamma}) \quad (0 < \gamma < \tfrac{1}{2}\dagger),$$
$$\partial f(x, y)/\partial x = O(f(x, y)),$$

where $f(x, y)$ is any quantity describing the hull geometry.

† The factor γ is introduced here because of problems arising later on in the transformation of the boundary condition.

The free surface is described by

$$G_2(x, y, z) = z - \zeta(x, y) = 0. \tag{3}$$

Besides the foregoing assumptions three others have to be made: that the fluid is ideal, that the flow is irrotational and that the problem can be linearized. The assumption of linearity of the problem is rather restrictive. However, the nonlinear problem is quite intractable and it is hoped that even the linearized solution can give useful information.

The boundary-value problem

The existence of a velocity potential

$$\Phi(x, y, z) = Ux + \phi(x, y, z) \tag{4}$$

is assumed. The continuity equation for an incompressible fluid then requires that

$$\frac{\partial^2\Phi}{\partial x^2} + \frac{\partial^2\Phi}{\partial y^2} + \frac{\partial^2\Phi}{\partial z^2} = 0 \quad \text{in the fluid region.} \tag{5}$$

The condition to be imposed on the wing surface is

$$D[G_0(x, y, z)]/Dt = 0 \quad \text{on} \quad G_0 = 0, \tag{6}$$

where the operator D/Dt stands for what is commonly called the substantial derivative.

On the free surface the dynamic and kinematic conditions to be satisfied are respectively

$$\left.\begin{aligned} g\zeta + \frac{1}{2}\left[\left(\frac{\partial\Phi}{\partial x}\right)^2 + \left(\frac{\partial\Phi}{\partial y}\right)^2 + \left(\frac{\partial\Phi}{\partial z}\right)^2\right] &= \text{constant} \\ D[G_2(x, y, z)]/Dt &= 0 \end{aligned}\right\} \quad \text{on } G_2 = 0. \tag{7), (8}$$

In addition a proper 'radiation condition' has to be imposed in order to ensure the uniqueness of the solution.

For the sake of completeness one should mention the necessity of imposing the Kutta condition, on the trailing edge. This condition will not be stated explicitly here since only the bow-region problem is studied.

The boundary-value problem (4)–(8) is nonlinear and will be linearized properly. For the solution of the linearized problem the method of matched asymptotic expansions is used, thus the existence of a near-field and a far-field region is assumed. In the far field as well as in the near field the potential is expressed in terms of an asymptotic expansion:

$$\Phi = \begin{cases} Ux + \varphi(x, y, z) & \text{in the far field,} \\ Ux + \phi(x, y, z) & \text{in the near field,} \end{cases}$$

where $\varphi(x, y, z)$ and $\phi(x, y, z)$ are perturbation potentials such that

$$\begin{aligned} \varphi(x, y, z) &= \varphi_1(x, y, z; \epsilon) + \varphi_2(x, y, z; \epsilon) + \ldots, \\ \phi(x, y, z) &= \phi_1(x, y, z; \epsilon) + \phi_2(x, y, z; \epsilon) + \ldots, \end{aligned}$$

and satisfy

$$\varphi_{n+1} = o(\varphi_n), \quad \phi_{n+1} = o(\phi_n) \quad \text{as} \quad \epsilon \to 0 \text{ with } x, y, z \text{ fixed.}$$

3. The solution of the problem

As mentioned above, the method of matched asymptotic expansions is used to solve the boundary-value problem presented in the previous section and the existence of a far-field as well as a near-field region is therefore assumed.

The far-field region

At distances from the ship equal to or greater than unity all the fine details of the flow are lost and only a gross disturbance caused by the ship hull can be seen; this is the far-field region. Here the disturbance created by a steadily turning ship can be represented by a line distribution of horizontal dipoles which have a constant density $\sigma(x)$ with respect to time. A lower-order singularity, i.e. a source distribution, is not considered here since the ship is assumed to be of zero thickness.

The Laplace equation must be satisfied in the fluid region on account of continuity. The conditions to be satisfied on the free surface are linearized in the usual way (Wehausen & Laitone 1960) and combined into a single condition to be satisfied on the plane of the undisturbed free surface:

$$U^2\partial^2\varphi/\partial x^2 + g\partial\varphi/\partial z = 0 \quad \text{on} \quad z = 0. \tag{9}†$$

Perhaps the easiest way to satisfy the radiation condition, which excludes the possibility of waves upstream of the ship, is to introduce the 'Rayleigh fictitious viscosity' μ, which is set equal to zero at an appropriate time (see Ogilvie & Tuck 1969). Condition (9) is then modified to

$$(U\partial/\partial x + \tfrac{1}{2}\mu)^2\varphi + g\varphi_z = 0 \quad \text{on} \quad z = 0. \tag{10}$$

The body boundary condition is not included here since it can be satisfied only in the near-field region.

A solution of the above boundary-value problem is given by Hirata (1972*b*) in the form

$$\varphi(x,y,z) = \frac{1}{2\pi}\int_{-\infty}^{\infty} dk\, e^{ikx}\sigma^*(k) \lim_{\mu\to 0}\left[-\frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{dl\, l \exp[ily + z(k^2+l^2)^{\frac{1}{2}}]}{(k^2+l^2)^{\frac{1}{2}} - g^{-1}(Uk - \frac{1}{2}i\mu)^2}\right], \tag{11}$$

where the asterisk indicates a Fourier transform, defined by

$$f^*(k) = \int_{-\infty}^{\infty} dx\, e^{-ikx} f(x).$$

The velocity potential (11) is not completely described as yet since the dipole density $\sigma(x)$ is still unknown. This density will be determined through the matching with the near-field solution.

The near-field region

In this region one wants to know the details of the flow near the body, and for that purpose the co-ordinates are stretched as indicated in § 1. It could therefore be assumed that

$$x = \epsilon^{\frac{1}{2}}X, \quad y = \epsilon Y, \quad z = \epsilon Z$$

† Only the first term in the perturbation potential is considered, therefore the subscript 1 will be omitted.

and
$$\partial/\partial X, \partial/\partial Y, \partial/\partial Z = O(1).$$

In fact, the above notation need not be used explicitly; instead, the following relations will be understood to hold in the near field:

$$x = O(\epsilon^{\frac{1}{2}}), \quad y, z = O(\epsilon), \quad \partial/\partial x = O(\epsilon^{-\frac{1}{2}}), \quad \partial/\partial y, \partial/\partial z = O(\epsilon^{-1}).$$

The fact that the derivative in the longitudinal direction is assumed to be $O(\epsilon^{-\frac{1}{2}})$ and the derivative in the transverse direction to be $O(\epsilon^{-1})$ means that two different scales are used in the near field, and that the variation in the flow quantities in the longitudinal direction is smaller than that in the transverse direction. Note that this is also true in the usual slender-body theory but there the derivative in the longitudinal direction is assumed to be $O(1)$, which seems to put too much emphasis on gravity effects; the present assumptions are made in order to de-emphasize these gravity effects near the bow, which seems to be realistic in view of observations and experiments.

Introducing the above assumptions into the boundary-value problem (4)–(8), after a straightforward expansion one gets†

$$\phi_{yy} + \phi_{zz} = 0 \quad \text{in the fluid region,} \tag{12}$$

$$\phi_{xx} + (g/U^2)\,\phi_z = 0 \quad \text{on} \quad z = 0, \tag{13}$$

$$\phi_y = Ub'(0) \quad \text{on} \quad y = \pm 0, \quad 0 > z > -h, \tag{14}$$

where only the leading terms have been included. The proper condition at infinity is given by the behaviour of (11) as one approaches the bow region.

In condition (14) it is assumed that a McLaurin series expansion for $b(x)$ exists, only the first term being considered. The interesting feature of this problem is the presence of a second derivative with respect to x in condition (13). This prevents the use of most complex-variable methods. However, by taking the Fourier transform in the x direction one gets a boundary-value problem in y and z having the transform variable k as a parameter.

Taking the x Fourier transform has certain implications: for instance, the body boundary condition gives
$$\phi_y = Ub'(0)$$
and the Fourier transform of ϕ_y is

$$\phi_y^* = \int_{-\infty}^{\infty} \phi_y(x, 0, z)\, e^{-ikx}\, dx,$$

i.e. ϕ_y^* contains information from $-\infty$ to $+\infty$. However, for $x < 0$ the approximation $\phi_y = 0$ on $y = 0$ seems to be reasonable; it is also known that in slender-body theory the influence of the flow downstream on the upstream region is of higher order. Therefore one can write approximately

$$\phi_y^* = \int_{-\infty}^{\infty} Ub'(0)\, e^{-ikx}\, dx = \int_{-\infty}^{\infty} Ub'(0)\, e^{-ikx} H(x)\, dx,$$

where $H(x)$ is the Heaviside step function. According to Lighthill (1964, p. 33), one has
$$\phi_y^* = Ub'(0)\,[\pi\delta(k) + 1/ik],$$
where $\delta(k)$ is the Dirac delta function.

† Note that the subscript 1 is again omitted.

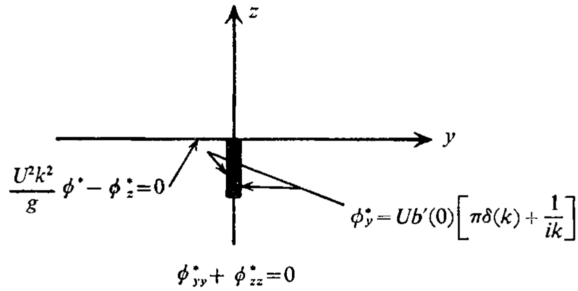

FIGURE 2. Two-dimensional boundary-value problem.

After taking the x Fourier transform of the boundary-value problem in the near field, one gets the following in the transformed plane:

$$\phi^*_{yy} + \phi^*_{zz} = 0 \quad \text{in the fluid domain,} \tag{15}$$

$$(U^2k^2/g)\,\phi^* - \phi^*_z = 0 \quad \text{on} \quad z = 0, \tag{16}$$

$$\phi^*_y = Ub'(0)\,[\pi\delta(k) + 1/ik] \quad \text{on} \quad y = \pm 0. \tag{17}$$

The resulting condition for $|y| \to \infty$ should match the inner expansion of (11).

The above problem is sketched in figure 2.

It can be shown (Hirata 1972*b*) that the behaviour of φ as one approaches the bow region is given by

$$\begin{aligned}\phi^* \sim \phi^*_{\pm\infty} = {} & [\pm\sigma^*_I(k)\,\alpha\, e^{\alpha z}\cos\alpha y - \sigma^*_R(k)\,\alpha\, e^{\alpha z}\,\mathrm{sgn}\,k\,\sin\alpha y] \\ & + i[\mp\sigma^*_R(k)\,\alpha\, e^{\alpha z}\cos\alpha y - \sigma^*_I(k)\,\alpha\, e^{\alpha z}\,\mathrm{sgn}\,k\,\sin\alpha y] \\ & \text{as} \quad y \to \pm\infty, \end{aligned} \tag{18}$$

where $\qquad \mathscr{F}[\sigma(x)] = \sigma^*(k) = \sigma^*_R(k) + i\sigma^*_I(k), \quad \alpha = U^2k^2/g.$

The elementary solution (18) satisfies (15), (16) and, of course, the condition for $|y| \to \infty$, but it is not possible to satisfy the body condition (17) using only combinations of (18). However, there is another elementary solution which satisfies (15) and (16) and decays to zero for $|y| \to \infty$:

$$\phi^*_A = e^{uy}(u\cos uz + \alpha\sin uz) \quad (y \geqslant 0), \tag{19}$$

where u is a positive real number.

One can attempt to write the solution to the boundary-value problem in the transformed plane, following Ursell (1947), as

$$\phi^* = \begin{cases}\phi^*_+ & (y \geqslant 0),\\ \phi^*_- & (y \leqslant 0),\end{cases} \tag{20}$$

where

$$\phi^*_\pm = \phi^*_{\pm\infty} + \int_0^\infty \{S(p) + iC(p)\}\, e^{\mp py}\{p\cos pz + \alpha\sin pz\} \quad (y \gtrless 0).$$

Let the body boundary condition be expressed as

$$\partial\phi^*/\partial y = f(z) + ig(z) \quad (y = 0),$$

with

$$f(z) = F_R, \quad g(z) = F_I, \quad F_R + iF_I = \mathscr{F}[\phi_y(x,0,z)] \quad (0 > z > -h).$$

Introducing (20) and using a lemma given by Ursell (1948) one gets

$$\sigma_R^* = \frac{-2}{\alpha \operatorname{sgn} k} \int_{-\infty}^{0} f(z)\, e^{\alpha z}\, dz, \tag{21}$$

$$\sigma_I^* = \frac{-2}{\alpha \operatorname{sgn} k} \int_{-\infty}^{0} g(z)\, e^{\alpha z}\, dz, \tag{22}$$

$$S(p) = \frac{-2}{\pi p(p^2+\alpha^2)} \int_{-\infty}^{0} f(z)\,[p \cos pz + \alpha \sin pz]\, dz, \tag{23}$$

$$C(p) = \frac{-2}{\pi p(p^2+\alpha^2)} \int_{-\infty}^{0} g(z)\,[p \cos pz + \alpha \sin pz]\, dz. \tag{24}$$

The continuity of the fluid at $y = 0$ requires that

$$\partial\phi_+^*/\partial y = \partial\phi_-^*/\partial y \quad (-h > z > -\infty),$$

and therefore

$$\phi_+^* = \phi_-^* \quad (-h > z > -\infty), \tag{25}$$

since $\phi_\pm^* \to 0$ as $z \to -\infty$.

Substituting (20) into (25) and using (21)–(24) one gets a pair of coupled integral equations:

$$-\pi e^{\alpha z} \operatorname{sgn} k \int_{-\infty}^{0} g(\xi)\, e^{\alpha\xi}\, d\xi = \int_{-\infty}^{0} du\, f(u) \times \int_0^\infty \frac{(p\cos pu + \alpha \sin pu)(p \cos pz + \alpha \sin pz)\, dp}{p(p^2+\alpha^2)}, \tag{26a}$$

$$\pi e^{\alpha z} \operatorname{sgn} k \int_{-\infty}^{0} f(\xi)\, e^{\alpha\xi}\, d\xi = \int_{-\infty}^{0} du\, g(u) \times \int_0^\infty \frac{(p\cos pu + \alpha \sin pu)(p \cos pz + \alpha \sin pz)\, dp}{p(p^2+\alpha^2)}. \tag{26b}$$

This pair of integral equations can be decoupled (Hirata 1972*b*) into two integral equations of the type

$$\int_h^\infty \frac{\nu(u)\, du}{v^2 - u^2} = \lambda(v) \quad (h < v < \infty)$$

for which the solution is known (Ursell 1947). Therefore the Fourier transform of the dipole density can be obtained as well as the functions $S(p)$ and $C(p)$.

In the next section a simple example is solved and some numerical results presented.

4. Free-surface elevation for a ship of constant draft

If the ship has a constant draft in the bow region, the calculations are greatly simplified. If only the leading term is taken, the dynamic free-surface condition gives the free-surface elevation near the ship as

$$\zeta = (-U/g)\, \phi_x(x,0,0),$$

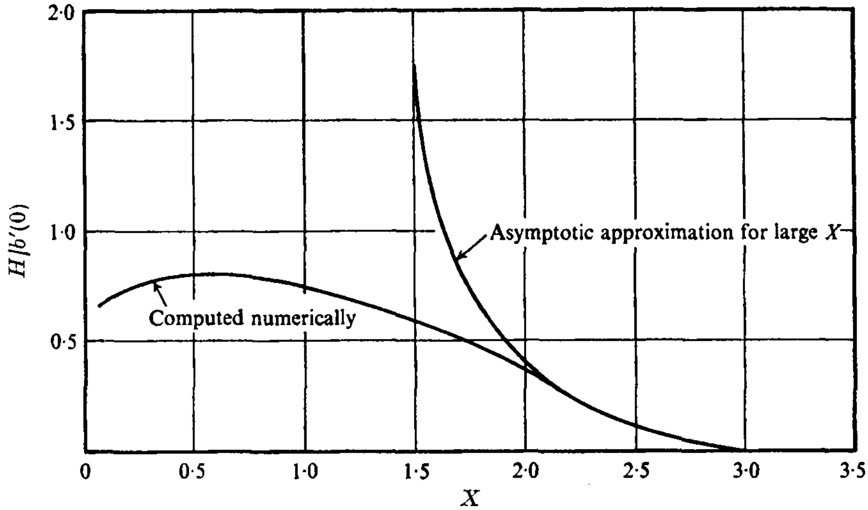

FIGURE 3. Non-dimensional free-surface elevation near the bow.

or
$$\zeta = -\frac{U}{g}\frac{i}{2\pi}\int_{-\infty}^{\infty} e^{ikx}\, k\phi^*(x,0,0)\, dk,$$
and using (18) one gets
$$\zeta(x,\pm 0) = \mp\frac{U}{g}\frac{i}{2\pi}\int_{-\infty}^{\infty} dk\, e^{ikx}\, k\{[\sigma_I^*\alpha - i\sigma_R\alpha] + \int_0^{\infty}[S(p)+iC(p)]\}\, p\, dp. \tag{27}$$

It can be shown (Hirata 1972*b*) that the contribution to the inverse Fourier transform when k is in the neighbourhood of zero is negligible, therefore one can neglect the contribution of $F_R = \pi\delta(k)$ in the body boundary condition. Using this fact, after decoupling the system of equations (26) the desired functions in (27) can be calculated:
$$-\alpha\sigma_R^* = \frac{\pi U b'(0)}{\alpha k}\,\frac{K_1(\alpha h)}{\pi^2 I_1^2(\alpha h) + K_1^2(\alpha h)}[I_1(\alpha h) + L_1(\alpha h)],$$
$$\alpha\sigma_I^* = \frac{\pi^2 U b'(0)\operatorname{sgn} k}{\alpha k}\,\frac{I_1(\alpha h)}{\pi^2 I_1^2(\alpha h) + K_1^2(\alpha h)} I_1(\alpha h) + L_1(\alpha h)],$$
$$\int_0^{\infty} S(p)\, p\, dp = \frac{U b'(0)\,\pi \operatorname{sgn} k}{\alpha k}\,\frac{\displaystyle\int_0^{\infty}\frac{pJ_1(ph)}{p^2+a^2}\,dp}{\pi^2 I_1^2(\alpha h) + K_1^2(\alpha h)}[I_1(\alpha h)\, L_1(\alpha h)],$$
$$\int_0^{\infty} C(p)\, p\, dp = -\frac{2Ub'(0)\,\alpha}{\pi k}\int_0^{\infty}\frac{\displaystyle\int_0^{ph} uJ_1(u)\, du}{p(p^2+\alpha^2)} + \frac{Ub'(0)}{k[\pi^2 I_1^2(\alpha h) + K_1^2(\alpha h)]}$$
$$\times\left[\frac{K_1(\alpha h)}{\alpha}\left(1 + \frac{2}{\pi}\int_0^{\alpha h} uK_1(u)\, du\right) + \frac{2\pi I_1(\alpha h)}{\alpha}\int_0^{\alpha h} uI_1(u)\, du\right]$$
$$\times\int_0^{\infty}\frac{pJ_1(ph)}{p^2+\alpha^2}\, dp,$$
where $I_1(x)$, $K_1(x)$ and $J_1(x)$ are Bessel functions and $L_1(x)$ is the modified Struve function (all defined according to Abramowitz & Stegun 1964, chap. 9).

For purposes of numerical calculation, the following non-dimensional coordinates are used:

$$X = \frac{x}{(hU^2/g)^{\frac{1}{2}}}, \quad K = k(hU^2/g)^{\frac{1}{2}}, \quad H = \frac{\zeta}{(h^2U/g)^{\frac{1}{2}}}.$$

Figure 3 shows a plot of $H/b'(0)$ *vs.* X. Equations (27) were used for the computations. This figure shows also an asymptotic estimate of the free-surface elevation for large X. This asymptotic estimate is of the form presented below and was obtained from (27) using a result from the theory of Fourier transforms (Lighthill 1964, theorem 19):

$$H/b'(0) \sim 12/X^5 \quad \text{as} \quad X \to \infty.$$

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